

Approximate Mathematical Model for load profiling and demand forecasting

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Abstract: This paper describes the development of approximate mathematical models to be used in load profiling and forecasting, e.i., the household and commercial load profile. The model was developed by studying and profiling load demand of Manyana, a village in Botswana. Basing on the random nonlinear regression results of the load demand profiles of Manyana village the numerical methods of least square approximation was chosen to determine the relationship between daily hours against energy consumption in kWh to get a mathematical representation between the two variables. The method of least square approximation enables approximate determination of the polynomial function of order n for any nonlinear regression discrete data. The work is still ongoing where by developed mathematical models will be used in coding to compute energy consumption prediction for any future energy demand and also generate a graphical representation for any other village in forecasting short, medium and long term energy demand.

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I. Introduction

The load profile give an indication of the energy consumption by users. The energy consumption could be that of a household, village, town or the entire country and could be expressed as per hour, day, week or year. It is important to determine the load profile and predict future energy demand for village(s) because accurate long term and midterm electric load forecasting plays an essential role in power system planning in terms of maintenance, construction scheduling for developing new power generation facilities, purchase of generating units and developing of transmission and distribution systems. It is difficult to forecast long term load demand accurately which means that we could overestimate the load demand which will result in substantial investment, that is higher construction, operation and maintenance costs of excess power facilities, whereas underestimating will result in customer dissatisfaction [2]. For utility companies, designers and policy makers to plan the efficient operation and economical expansion of an electric power delivery system, they must be able to know how much power must be delivered, where and when it will be needed, that information is provided by a load forecast which is defined as a prediction of future energy demand which includes the location, magnitude (how much power is needed) and when it is needed (temporal characteristics) [8]. While the national load profile particularly in Botswana shows peak demand around 6.00 a.m. to 10.00 a.m., and also from 6.00 p.m. to 10.00 p.m., as shown in the figure 1 [5], it is also necessary to consider villages independently when planning for the power supply because some villages without heavy industrial base and predominantly having household only would have different demand profile than those with heavy industry and commercial base.

Load forecast also helps us to extend our transmission lines to other neighbouring independent power producers so as to import if we are in deficit [5]. Under estimating the load forecast will result in severe impacts such as power deficit which means they will spend more in importing electricity from other countries and possible power cuts possibly resulting in blackouts in the entire country. On the other hand overestimating could be costly if the country is having a surplus with no market to sell to. Therefore, load profiling and forecasting help the owner of the power delivery system to schedule additional equipment and facilities to meet the anticipated demand for increased power consumption [8]. As a result it is necessary to have accurate load profiling and forecasting programme which can effectively be used to determine future energy demand. Accurate load model is required to mathematically represent the relationship between the load and influential variables including time, weather, seasons and socio economic factors. The exact relationship between the load and this variables is determined by their role in the load model. Furthermore, after the construction of a mathematical model, the model parameters can be determined by the use of estimation techniques such as qualitative analysis, time series, drift methods and other simulation methods. Because of factors such as weather and economic indices which are difficult to predict accurately for longer lead times ahead it means that the shorter the lead time, the more accurate the prediction is likely to be [7].

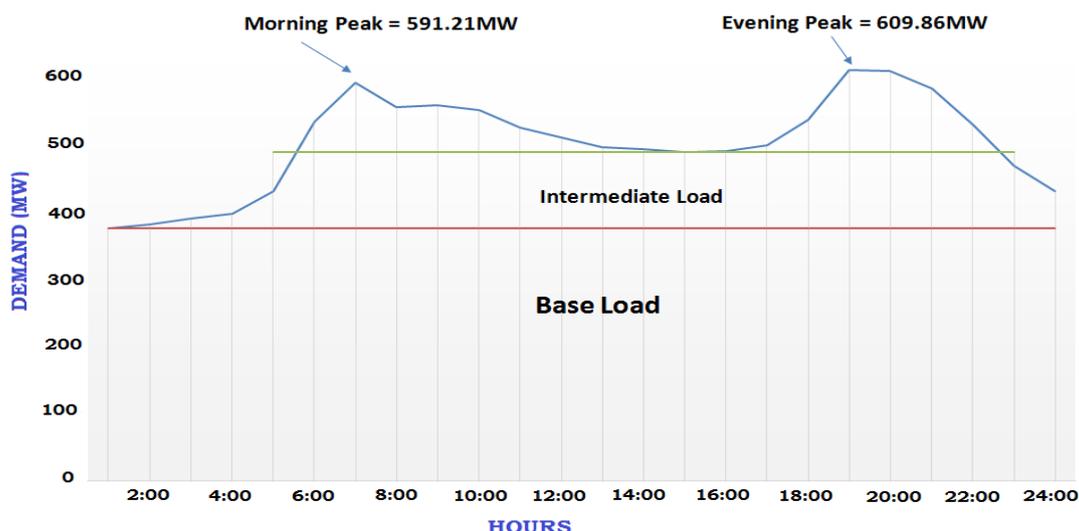


Figure 1. Converter Typical Hourly System Load

The rest of the paper is divided as follows: Section II outlines the reasons which necessitated the development of this proposal. Section III gives a detailed methodology of coming up with mathematical models including methods of determining polynomial function of the order n. Section IV gives the results and analysis while Section V gives conclusion and future work.

II. Load Forecasting

There are various approaches applied to load forecasting which ranges from regression-based models, classical time series and regression methods, over time-series approaches, hybrid artificial intelligence and computational intelligence methods. The accuracy of forecasts is a very crucial factor in load forecasting hence a decision maker in the energy sector has the need of accurate forecasts because most of the decisions are necessarily based on forecasts of future demands. The first decision to be made is therefore the selection of an appropriate model and this depends on the problem and the situation currently under consideration. It is in this case necessary to come up with a model which could be used under this instance. To do this, a survey was conducted in Manyana, which is a village distant away from industrialised cities to determine pattern of usage of electrical appliances by households and commercial enterprise. The information from the survey was used as part of developing the mathematical model.

III. Least Square Approximation Method

For this research the method of least square approximation method was used. The method of least square approximation allowed to approximately determine the polynomial function of order n .for any nonlinear regression discrete data [6]. A relationship between daily hours against power consumption in kW was determined and a mathematical representation between the two variables was obtained. The relationship between the daily hours and energy consumption help in generating a software code to simulate different scenarios in determining the load profiles and load forecasting of any chosen villages. Consider Table 1 below, which shows two variables X and Y. and generate a relationship between the two as discussed in [6].

Table 1. Showing variables X and Y

X_0	X_1	X_2	X_3	X_m
Y_0	Y_1	Y_2	Y_3	Y_m

The relationship helps to derive the polynomial function that is used to approximate and simulate load profiles and load forecasting for any village by different inputs scenarios.

$$P_n(X) = a * 0 + a * 1X + 2X^2 + a * 2X^2 + a * 3X^3 + \dots + a * nX^n \dots\dots\dots(1)$$

For which the function

$$D(a_0, a_1, a_2, \dots, a_n) = [Y_0 - P_n(X_0)]^2 + [Y_1 - P_n(X_1)]^2 + [Y_2 - P_n(X_2)]^2 + \dots + [Y_{n_1} - P_n(X_n)]^2$$

attains its minimum at $a * 0, a * 1, a * 2, \dots, a * n$ in the whole real space R^{n+1}

Then:

The function $D(a_0, a_1, a_2, \dots, a_n)$ is a non-negative quadratic function in the whole real space, therefore attains its minimum only if the conditions below are satisfied, [6]

$$\frac{\partial yD(a_0, a_1, a_2, \dots, a_n)}{\partial a_k} = 0, \quad k = 0, 1, 2, \dots, n$$

$$\frac{\partial yD(a_0, a_1, a_2, \dots, a_n)}{\partial a_k} = 2a_n \sum_{i=0}^m x^{k+n} = 0$$

Hence the following equation is obtained:

$$a_0 \sum_{i=0}^m xi + a_1 \sum_{i=0}^m xi^{k+1} + a_2 \sum_{i=0}^m xi^{k+2} + \dots + a_n \sum_{i=0}^m xi^{k+n} = \sum_{i=0}^m xi^k yi \quad \text{for } k = 0, 1, 2, 3, \dots, n, \quad (2)$$

The following polynomials are generated:

$$S_{0k} = m+1 \dots \dots \dots (3)$$

$$S_k = \sum_{i=0}^m xi^k \dots \dots \dots (4)$$

$$V_0 = \sum_{i=0}^m yi \dots \dots \dots (5)$$

$$SV_k = \sum_{i=0}^m x^k yi \dots \dots \dots (6)$$

To solve for the unknown polynomials given in equations 3 to 6, equations 2 can be written as:

$$S_0 a_0 + S_1 a_1 + S_2 a_2 + S_3 a_3 + \dots + S_n a_n = V_0$$

$$S_1 a_1 + S_2 a_2 + S_3 a_3 + S_4 a_4 + \dots + S_n a_n = V_1$$

$$S_2 a_2 + S_3 a_3 + S_4 a_4 + S_5 a_5 + \dots + S_n a_n = V_2$$

$$S_0 a_0 + S_1 a_1 + S_2 a_2 + S_3 a_3 + \dots + S_n a_n = V_0$$

....

$$S_0 a_0 + S_1 a_1 + S_2 a_2 + S_3 a_3 + \dots + S_n a_n = V_0$$

$$S_n a_0 + S_{n+1} a_1 + S_{n+2} a_2 + S_{n+3} a_3 + \dots + S_{2n} a_n = V_n \dots \dots \dots (7)$$

Solving the system of equations in (7) using matrices gives unique solution in the form $(a^*_0, a^*_1, a^*_2, \dots, a^*_n)$. And substituting the above solution in the polynomial in (1), the root mean square error is then estimated using the formula:

$$E = \sqrt{\frac{D(a_0, a_1, a_2, \dots, a_n)}{m + 1}} \dots \dots \dots (8)$$

IV. Analysis And Discussion Of The Results

(a) Survey results

Information was collected by managing a questionnaire which was developed specifically for the exercise. It covered all the electrical appliances found and used in households and commercial places, and the time of use. The kW and/ or kWh ratings was also recorded. From data collected, energy consumption for different seasons was calculated as in Table 2 below. The Excel Spreadsheet was used to generate graphs using data in Table 2.

Table 2. Energy consumption for a typical household

time (hours)	HOUSEHOLD PROFILE		COMMERCIAL PROFILE		TOTAL VILLAGE PROFILE	
	summer energy consumption (kwh))	winter energy consumption (kwh)	summer energy consumption (kwh)	winter energy consumption (kwh)	summer energy consumption (kwh)	winter energy consumption (kwh)
0.00	5.224	4.809	12.90825	15.73052	18.13225	20.53952
1.00	2.904	4.104	11.367	14.0054	14.271	18.1094
2.00	2.906	4.204	11.367	14.0054	14.273	18.2094
3.00	2.804	4.404	11.367	14.0054	14.171	18.4094
4.00	3.14	3.804	11.367	14.0054	14.507	17.8094
5.00	5.574	6.974	11.367	14.0054	16.941	20.9794
6.00	6.868	10.77	11.367	14.0054	18.235	24.7754
7.00	8.625	11.094	11.367	14.0054	19.992	25.0994
8.00	3.824	6.144	13.767	17.2631	17.591	23.4071
9.00	4.793	3.147	14.567	19.8031	19.36	22.9501
10.00	4.614	4.294	22.217	25.7121	26.831	30.0061
11.00	4.954	5.045	23.1292	27.4461	28.0832	32.4911

Table 2. (continues)

time (hours)	HOUSEHOLD PROFILE		COMMERCIAL PROFILE		TOTAL VILLAGE PROFILE	
	summer energy consumption (kwh)	winter energy consumption (kwh)	summer energy consumption (kwh)	winter energy consumption (kwh)	summer energy consumption (kwh)	winter energy consumption (kwh)
12.00	4.854	5.39	23.4292	22.6261	28.2832	28.0161
13.00	4.224	4.876	22.8592	22.7261	27.0832	27.6021
14.00	5.698	5.634	19.8592	22.6261	25.5572	28.2601
15.00	4.088	4.884	25.3192	22.6861	29.4072	27.5701
16.00	4.624	5.475	23.3192	22.6861	27.9432	28.1611
17.00	4.584	4.855	23.3192	22.6861	27.9032	27.5411
18.00	5.161	4.407	22.9192	22.6231	28.0802	27.0301
19.00	5.531	5.831	23.78045	24.93852	29.31145	30.76952
20.00	10.06	12.3695	23.73045	24.93852	33.79045	37.30802
21.00	6.45	6.259	13.82045	15.73052	20.27045	21.98952
22.00	6.12	6.19	12.90825	15.73052	19.02825	21.92052
23.00	7.279	7.349	12.90825	15.73052	20.18725	23.07952

(b) Comparison between the household load profiles in summer and winter

Figure 2 shows a non-linear trend between energy consumption and time in four hours. A morning energy peak demand from 0400 to 0600 am with a maximum peak of 11.094kWh and 8.625 kWh for winter and summer respectively. The energy demand peak rises again between 1800 and 2000 hours with a maximum peak demand of 10.06 and 12.3695 kWh for summer and winter respectively.

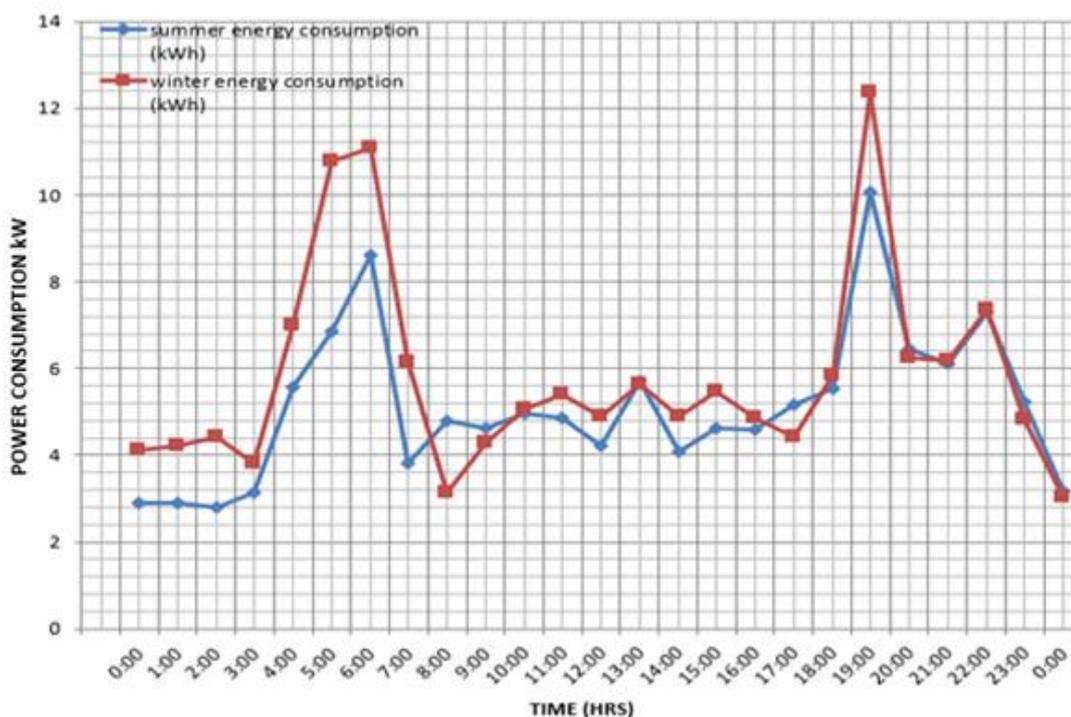


Figure 2. Comparison between the household load profiles in summer and winter

(c) Comparison between the commercial load profiles in summer and winter

The commercial load profile in Figure 3 below displays a nonlinear relationship between energy consumption and daily hours. It displays a peak demand generally from 0800 to 1900 hours for both winter and summer with maximum peak of 25.30 kWh and 27.44 kWh for summer and winter respectively.

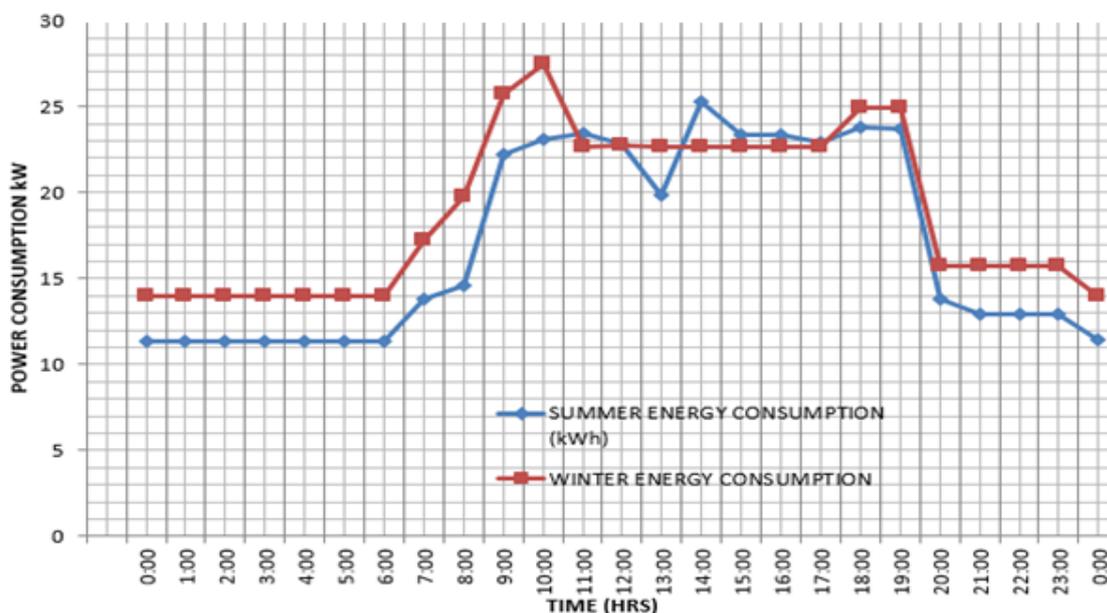


Figure 3. Commercial load profile in summer and winter

(d) Comparison summer and winter load profiles of combined commercial and household

Combining the commercial and households load profiles yields the total load profile for the entire village for both summer and winter as shown in Figure 4. There exists a nonlinear relationship between the two variables with winter load profile showing peak energy demand from 0800 and 1000 hours with a maximum morning peak of 28.0161kWh and another evening peak demand between 1800 and 2000 hours with a maximum evening peak of 37.308kWh.

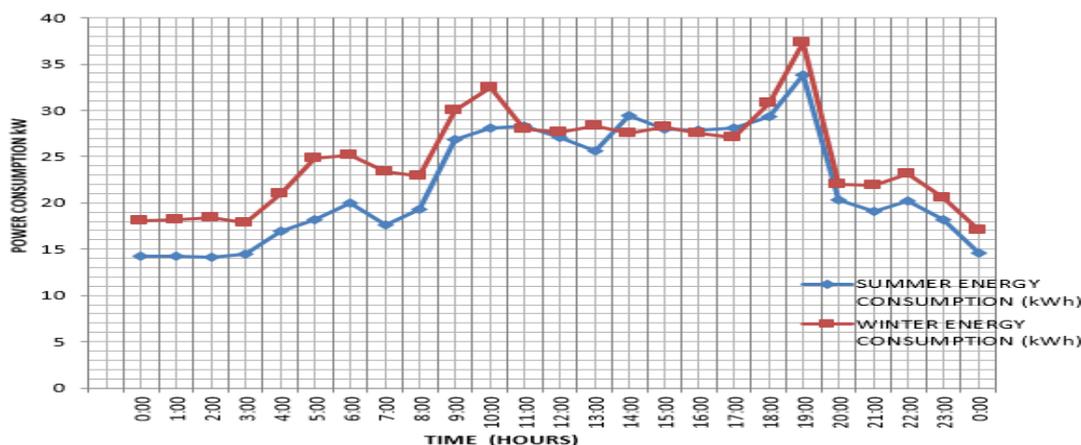


Figure 4. Combined commercial and household load profile in summer and winter

Using a similar approach all the polynomials we determined and they are as follows:

(a) Commercial summerload profile- mathematical model:

$$P_{SUM_COM}(t) = (0.0000t^6 - 0.0002t^5 + 0.0042t^4 - 0.0777t^3 + 0.9109t^2 - 3.6664t + 14.9572) + E$$

(b) Commercial winter load profile- mathematical model:

$$P_{com_win}(t) = (-0.0006t^5 + 0.0182t^4 - 0.3008t^3 + 2.5158t^2 - 8.2222t + 21.2101) + E$$

(c) Household winter load profile- mathematical model

$$P_{hous_win}(t) = (0.0000t^6 - 0.0005t^5 + 0.0124t^4 - 0.1380t^3 + 0.4834t^2 + 0.8713t + 2.1447) + E$$

(d) Household summer load profile- mathematical model

$$P_{sumer}(t) = (-0.0005t^5 + 0.0124t^4 - 0.1380t^3 + 0.4834t^2 + 0.8713t + 2.1447) + E$$

V. Conclusion And Future Work

In figures 2, 3 and 4, the demand for energy was high during winter which showed that there is energy is used in large quantities which is a fairly realistic situation because during winter, people switch on loads such as a electric geysers, air-conditioners, and other loads which draw a lot of power. The developed mathematical models also show that they can be used to determine load profiles of other villages. .

The work is still ongoing, the load profile of other village(s) will be determined using developed mathematical models. To expand the work factor, some other variables will be incorporated in the model to determine how the load profile is affected. The variables include but not limited to socio economic factors, such as at those rural villages where some people migrate to ploughing areas during ploughing season.

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